A General-Purpose Counting Filter: Making Every Bit Count

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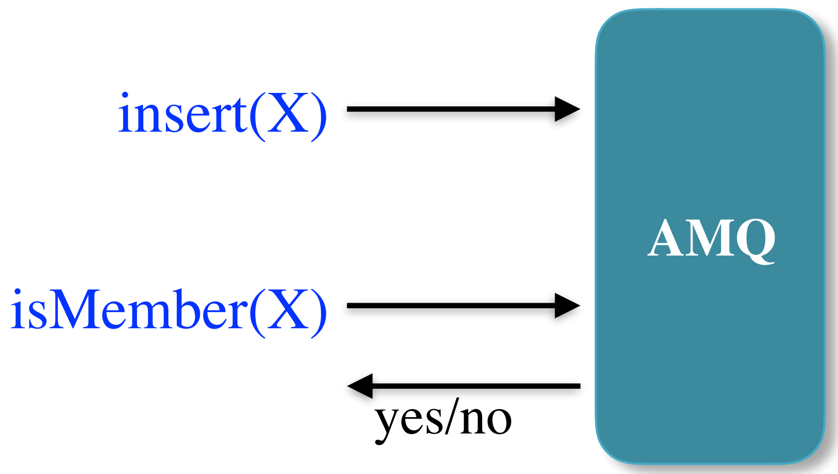
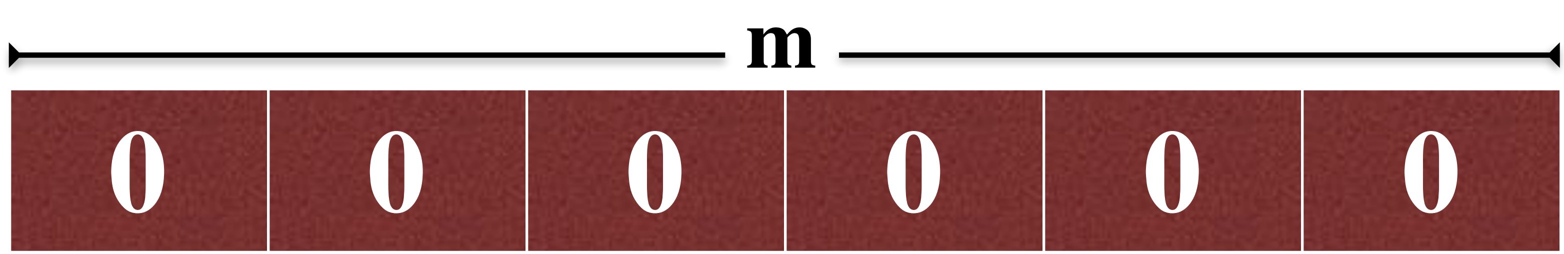
Approximate Membership Query
(AMQ)



- **An AMQ is a lossy representation of a set.**
- **Operations: inserts and membership queries.**
- **Compact space:**
  - Often taking < 1 byte per item.
  - Comes at the cost of occasional false positives.

Bloom filter

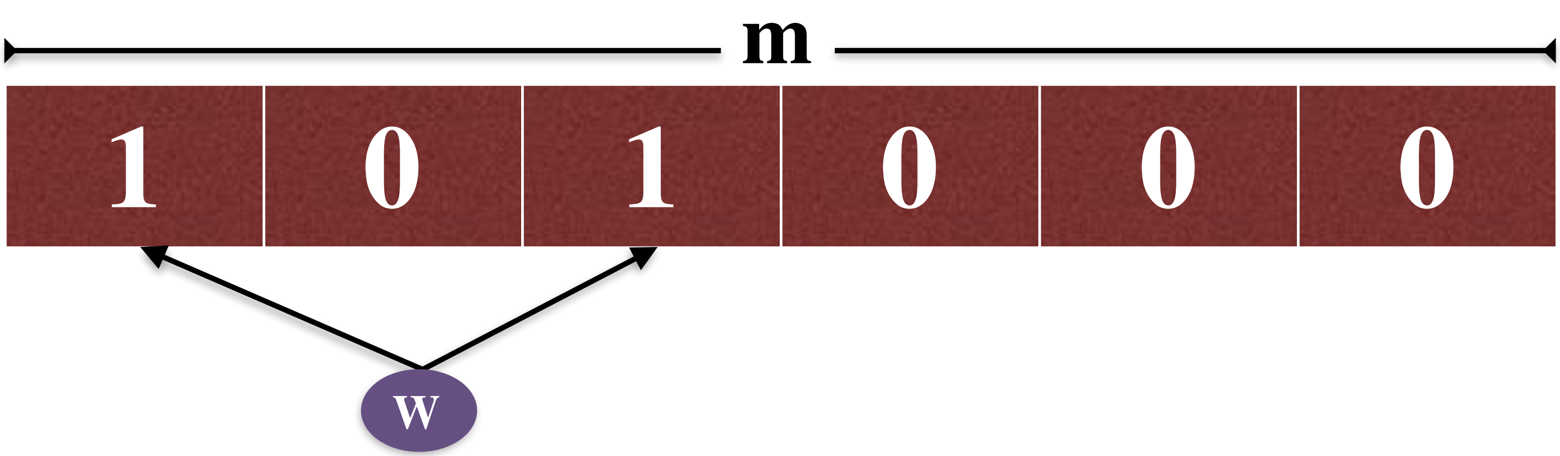
[Bloom, 1970]



- A Bloom filter is a bit-array + k hash functions.

  (Here k=2.)

Insertions in a Bloom filter



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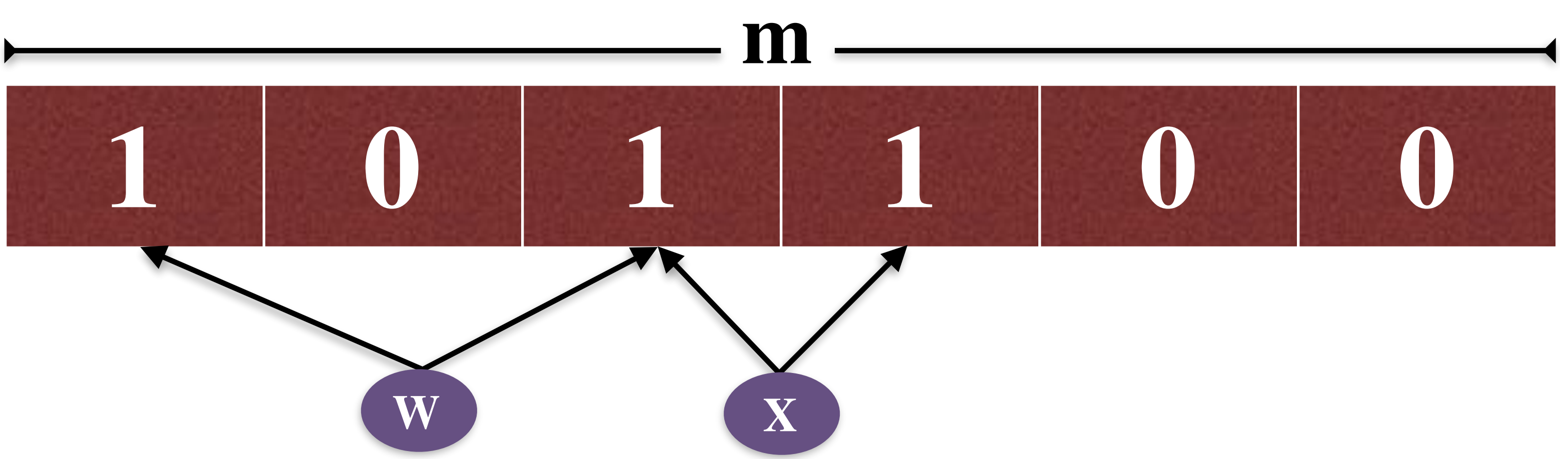

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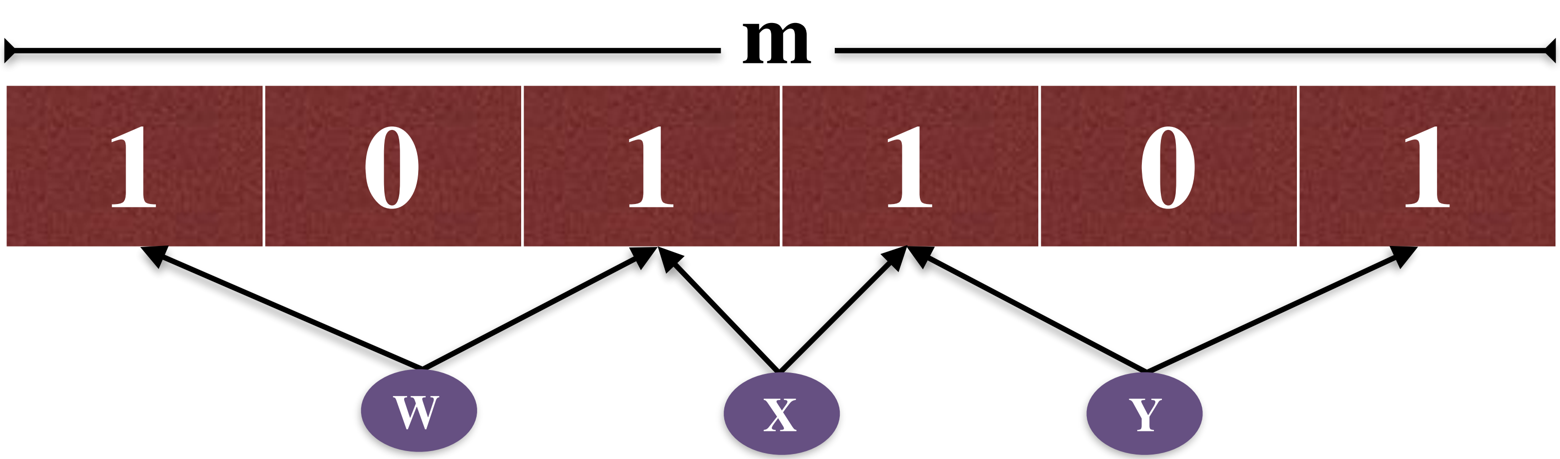

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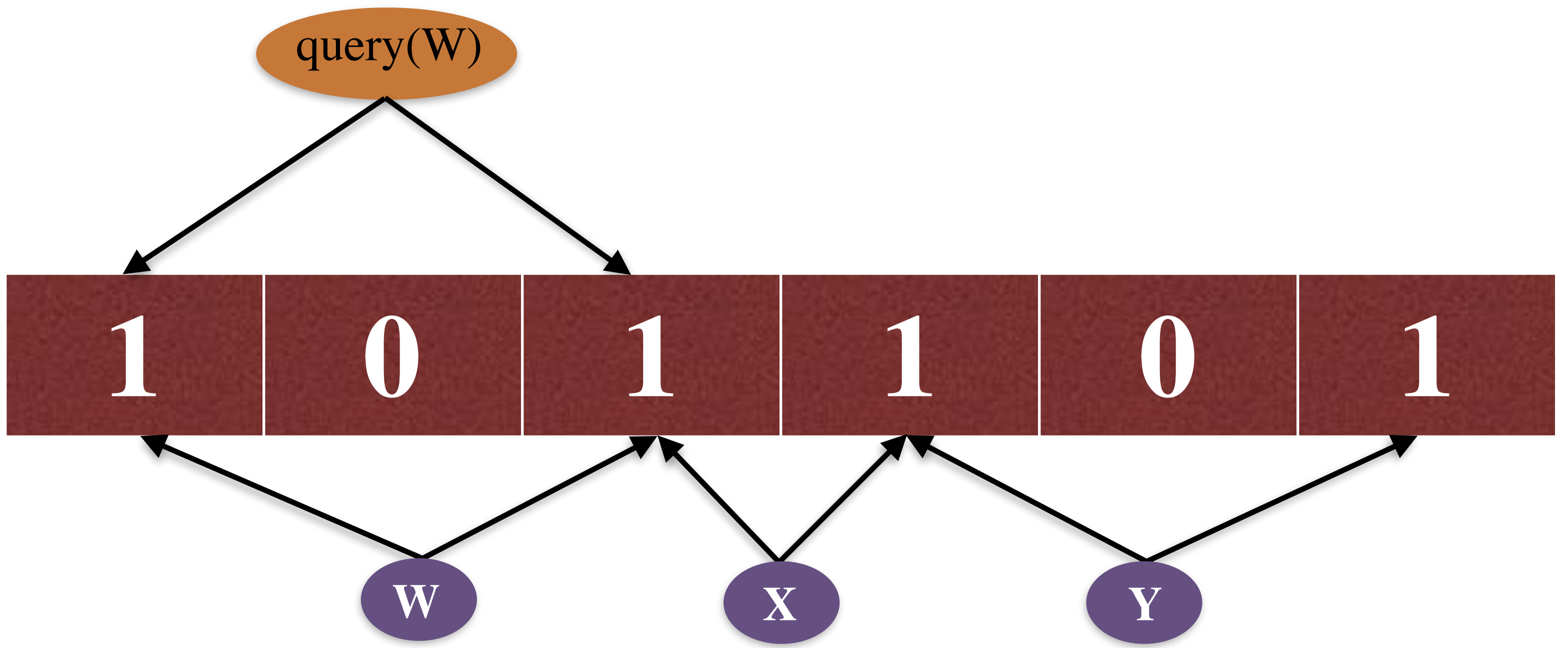
Membership query in a Bloom filter



- The Bloom filter has a bounded false-positive rate.

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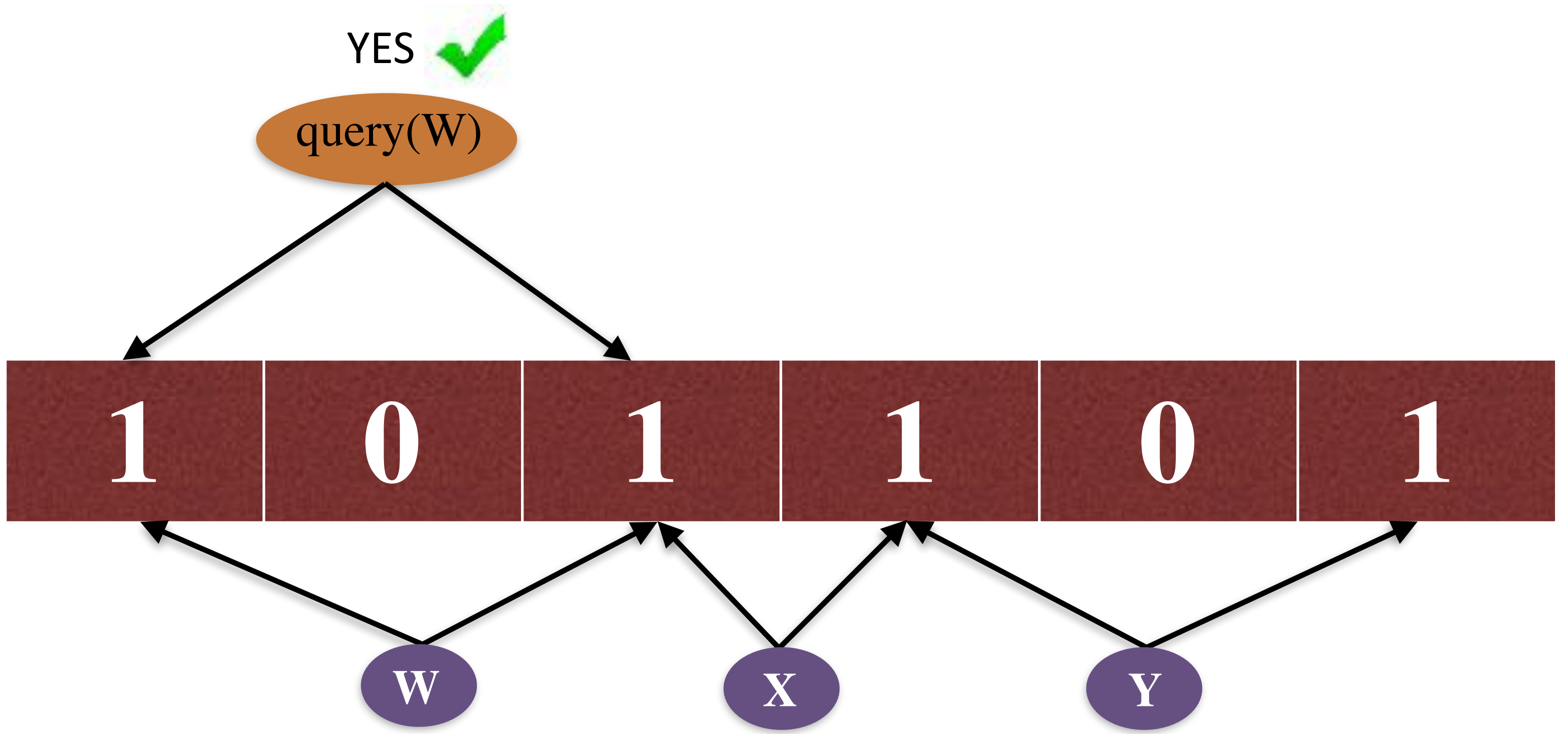

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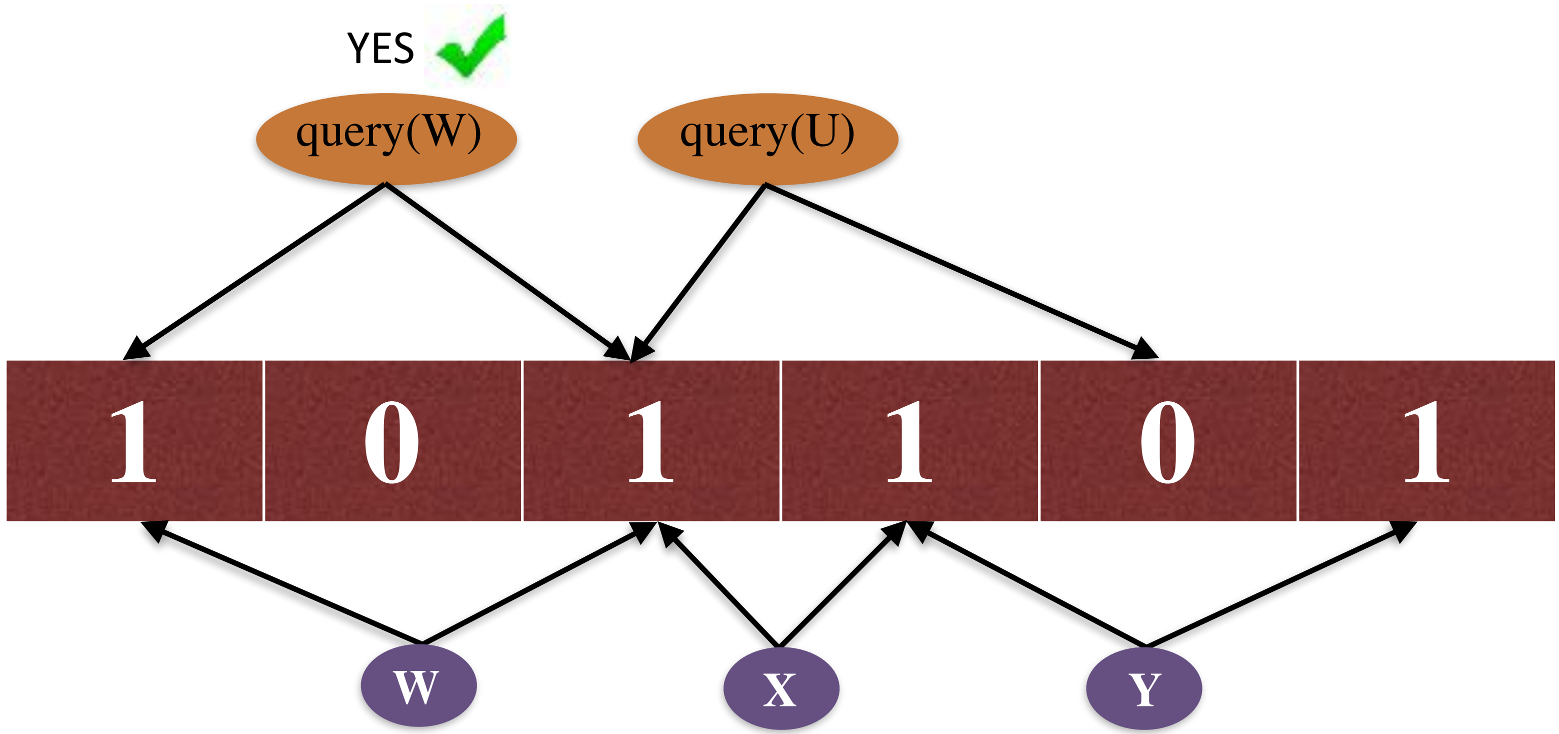

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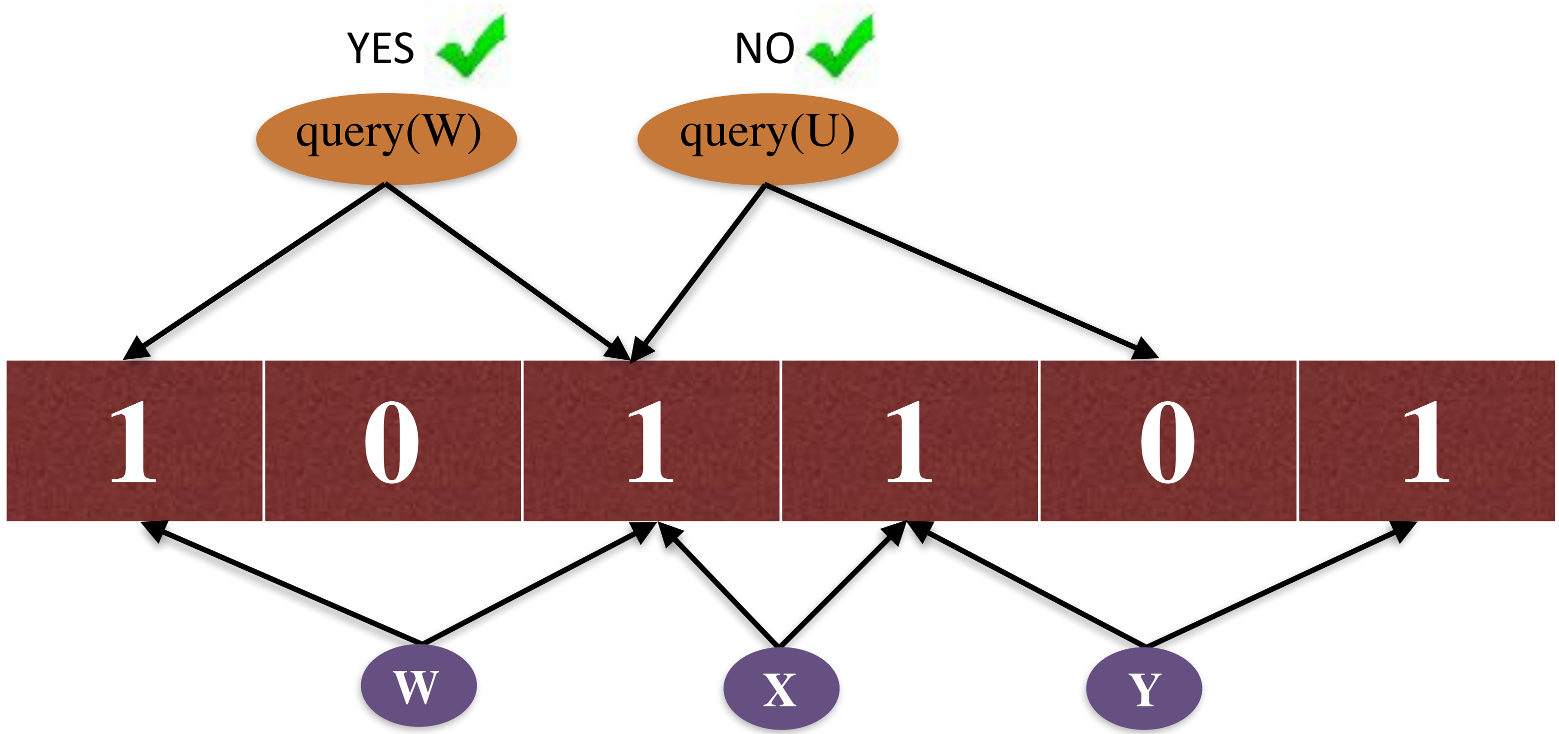

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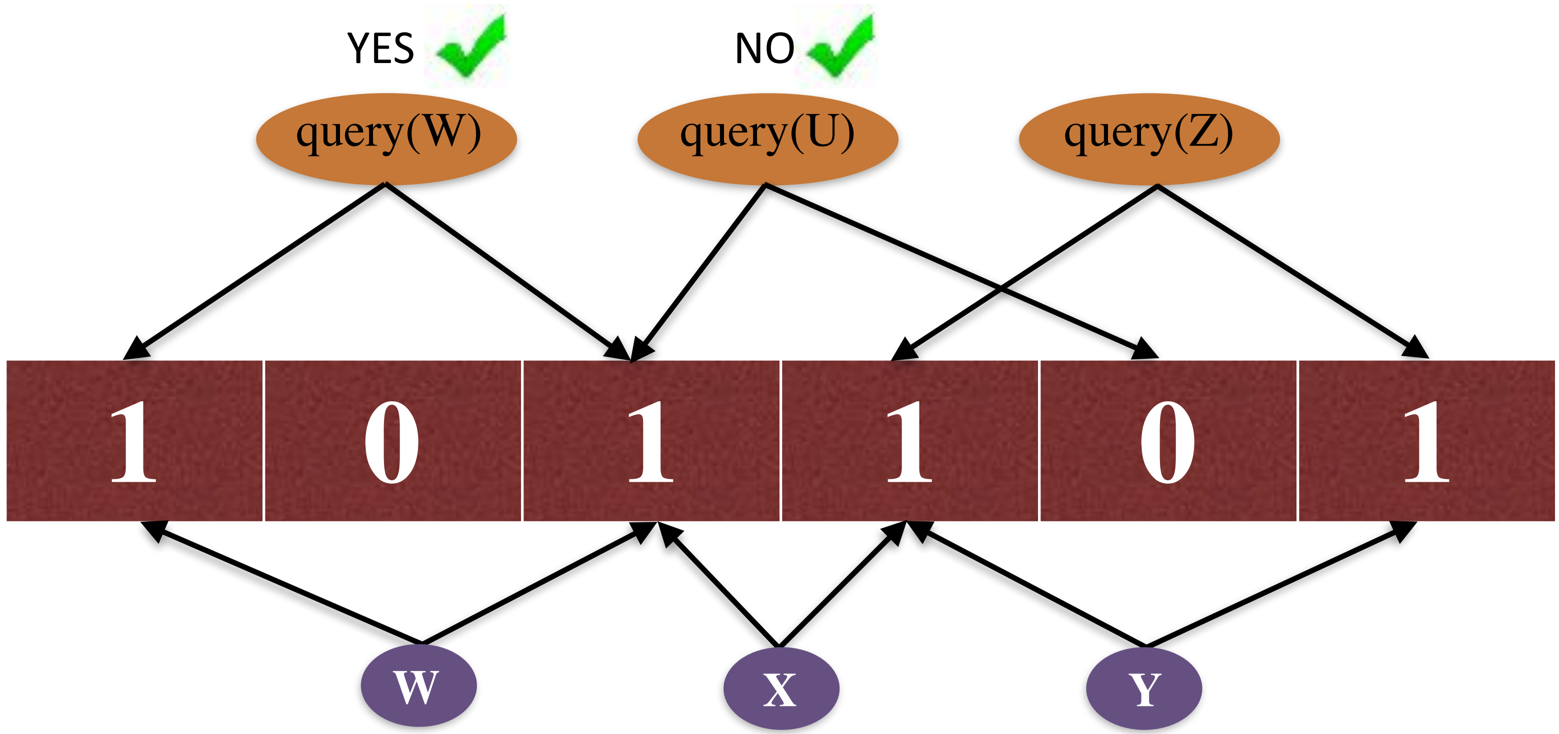

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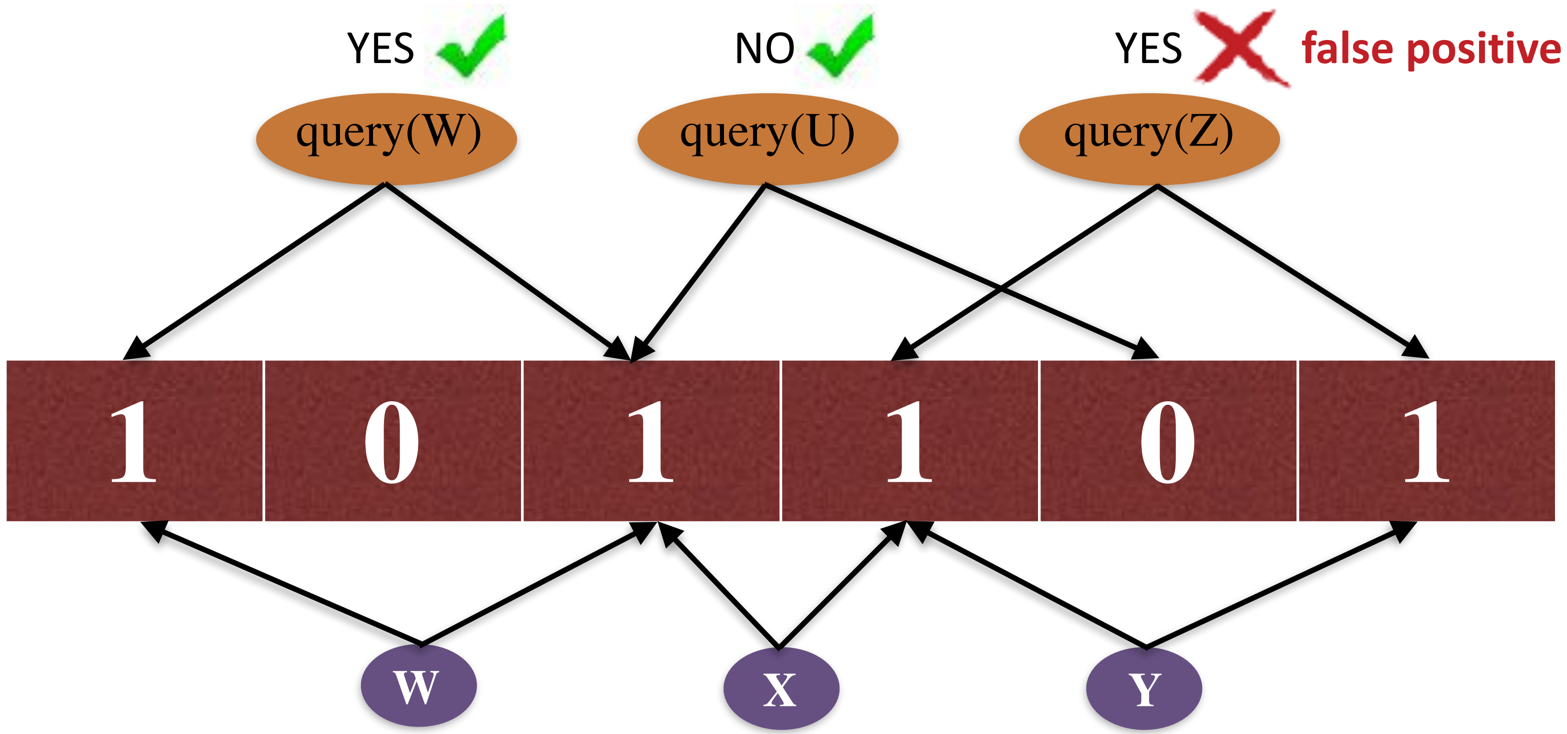

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Bloom filters are ubiquitous

Streaming applications

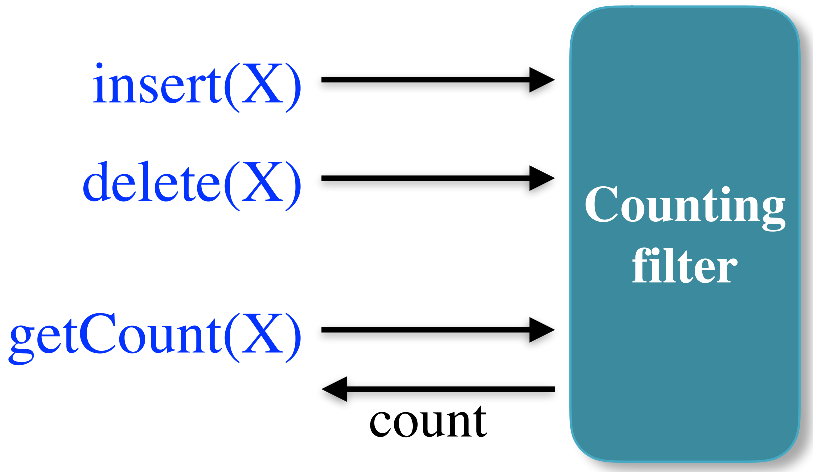
Networking

Databases

Computational biology

Storage systems

Counting filters: AMQs for multisets

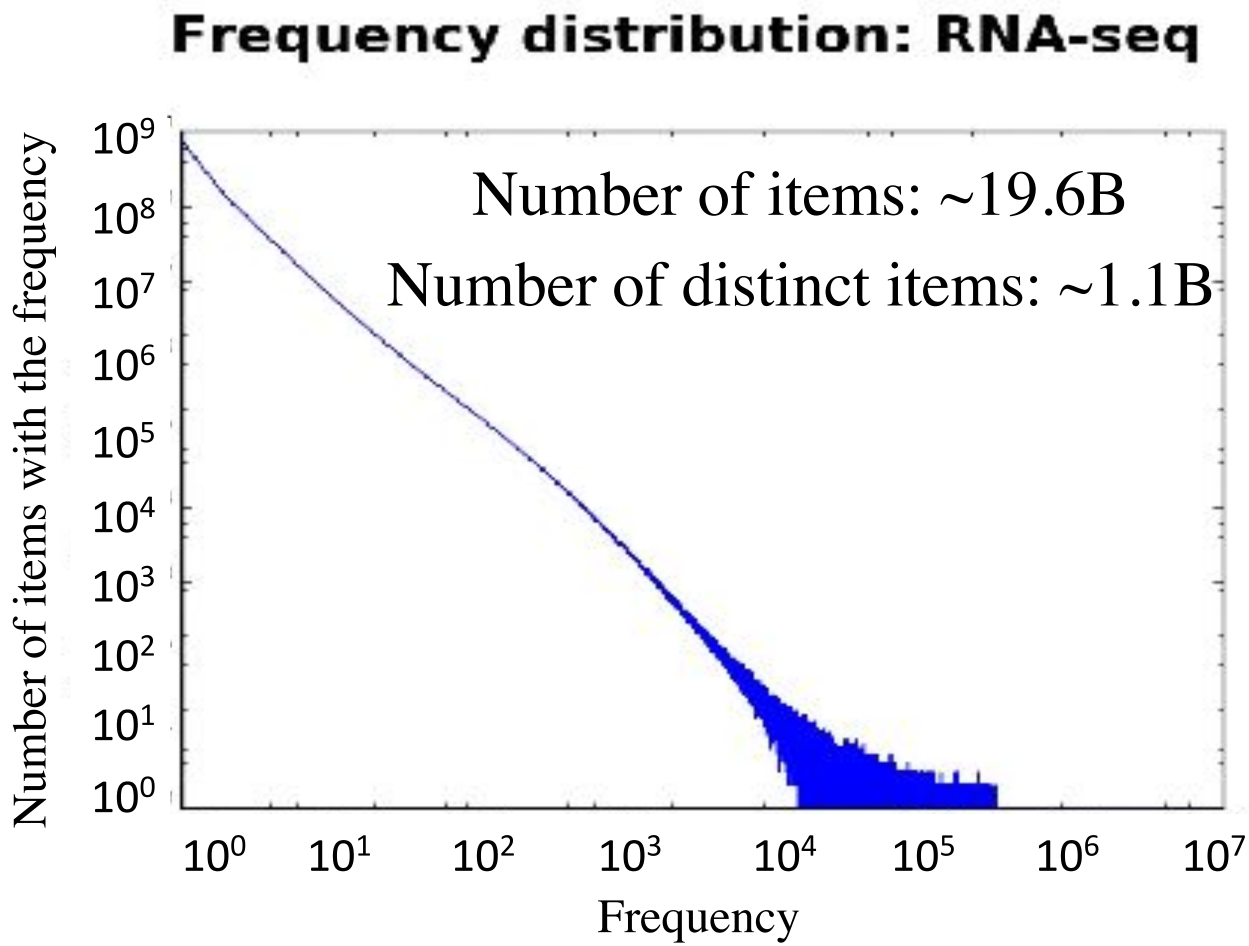


- **A counting filter is a lossy representation of a multiset.**
- **Operations: inserts, count, and delete.**
- **Generalizes AMQs**
    - False positives ≈ over-counts.

Why is counting important?

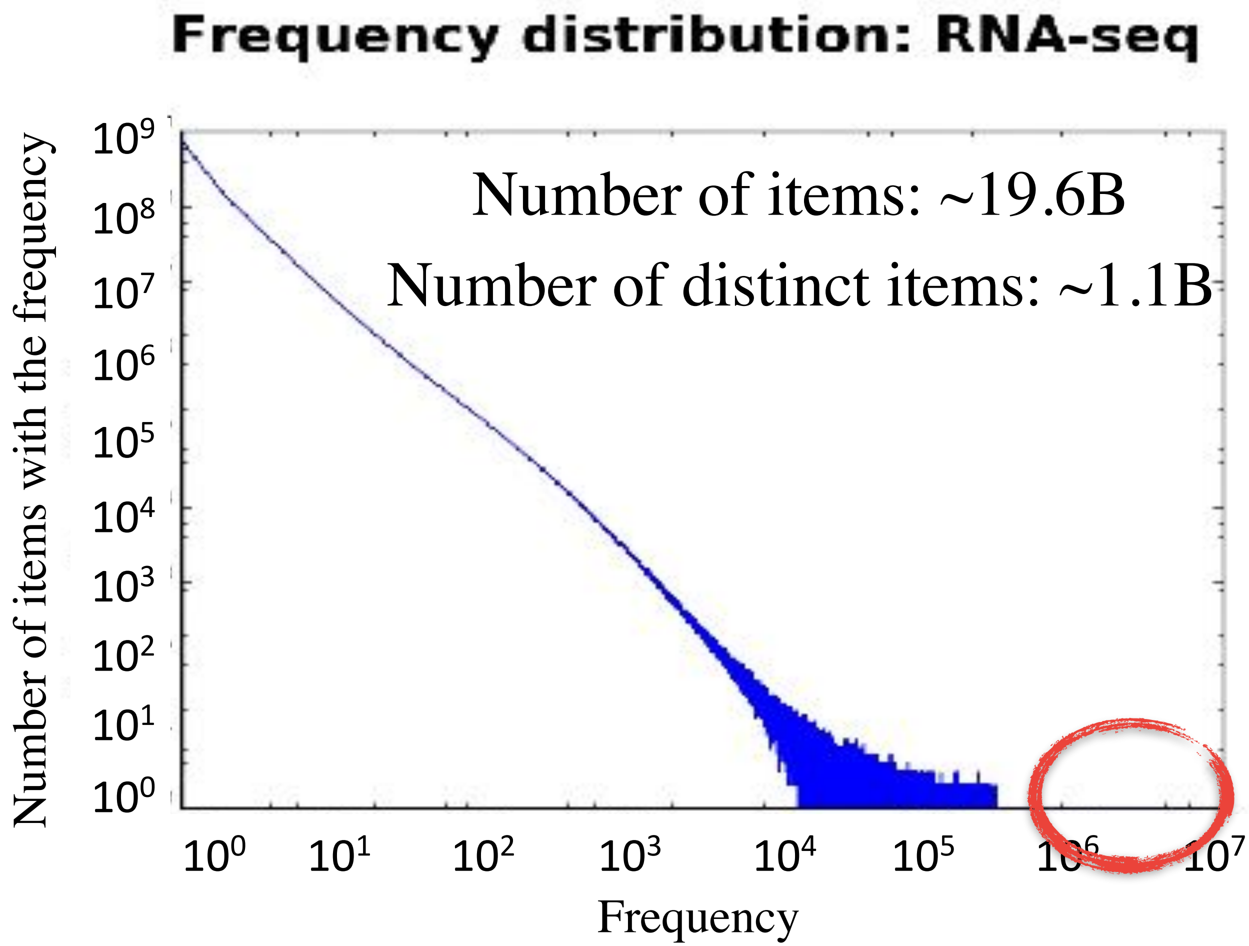
- Counting filters have numerous applications:
  - Computational biology, e.g., k-mer counting.
  - Network anomaly detection.
  - Natural language processing, e.g., n-gram counting.
- Counting enables AMQs to support deletes.

Many real data sets have skewed counts



Counting filters should handle skewed data sets efficiently.

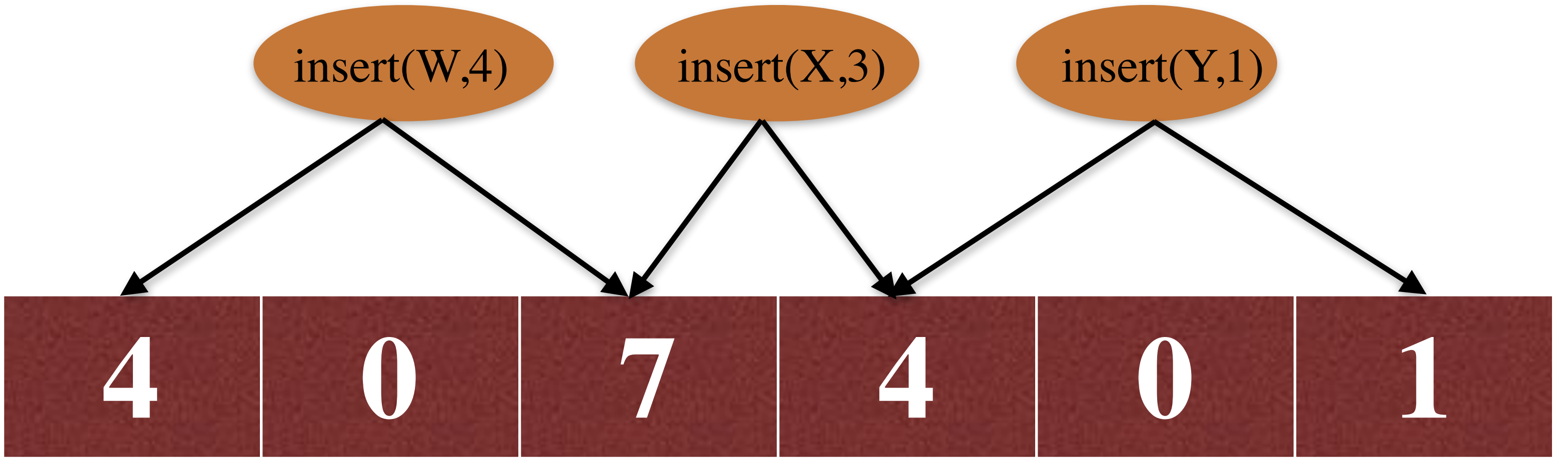
Many real data sets have skewed counts



Counting filters should handle skewed data sets efficiently.

Counting Bloom filters

[Fan et al., 2000]



- Counters must be large enough to hold count of most frequent item.
- Counting Bloom filters are not space-efficient for skewed data sets.

Counting Bloom filters

[Fan et al., 2000]

RNA-seq dataset

Total number of items: 19.6 Billion

Number of distinct items: 1.1 Billion

Maximum frequency: ~8 Million

Space usage of a CBF: ~38GB

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- Counting Bloom filters are not space-efficient for skewed data sets.

This paper: The counting quotient filter (CQF)

- A replacement for the (counting) Bloom filter.
- Space and computationally efficient.
- Uses variable-sized counters to handle skewed data sets efficiently.

$$\text{CQF space} \leq \text{BF space} + \underbrace{O\left(\sum_{x \in S} \log c(x)\right)}_{\text{Asymptotically optimal}}$$

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RNA-seq dataset

Total number of items: 19.6 Billion

Number of distinct items: 1.1 Billion

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Space usage of a CQF: ~2.5GB

$$\text{CQF space} \leq \text{BF space} + O(\underbrace{\sum_{x \in S} \log c(x)}_{\text{Asymptotically optimal}})$$

Other features of the CQF

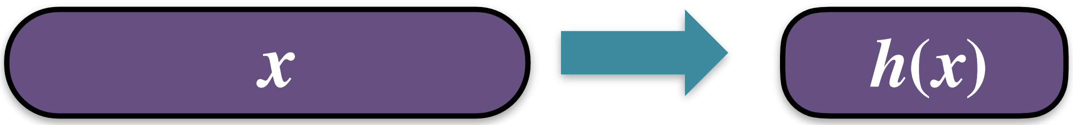
- Smaller than many non-counting AMQs
  - Bloom, cuckoo [Fan et al., 2014], and quotient [Bender et al., 2012] filters.
- Good cache locality
- Deletions
- Dynamically resizable
- Mergeable

Contributions

- New quotient filter metadata scheme
    - Smaller and faster than original quotient filter
- Efficient variable-length counter encoding method
    - Zero overhead for counters
- Fast implementation of bit-vector select on words
    - Exploits new x86 bit-manipulation instructions

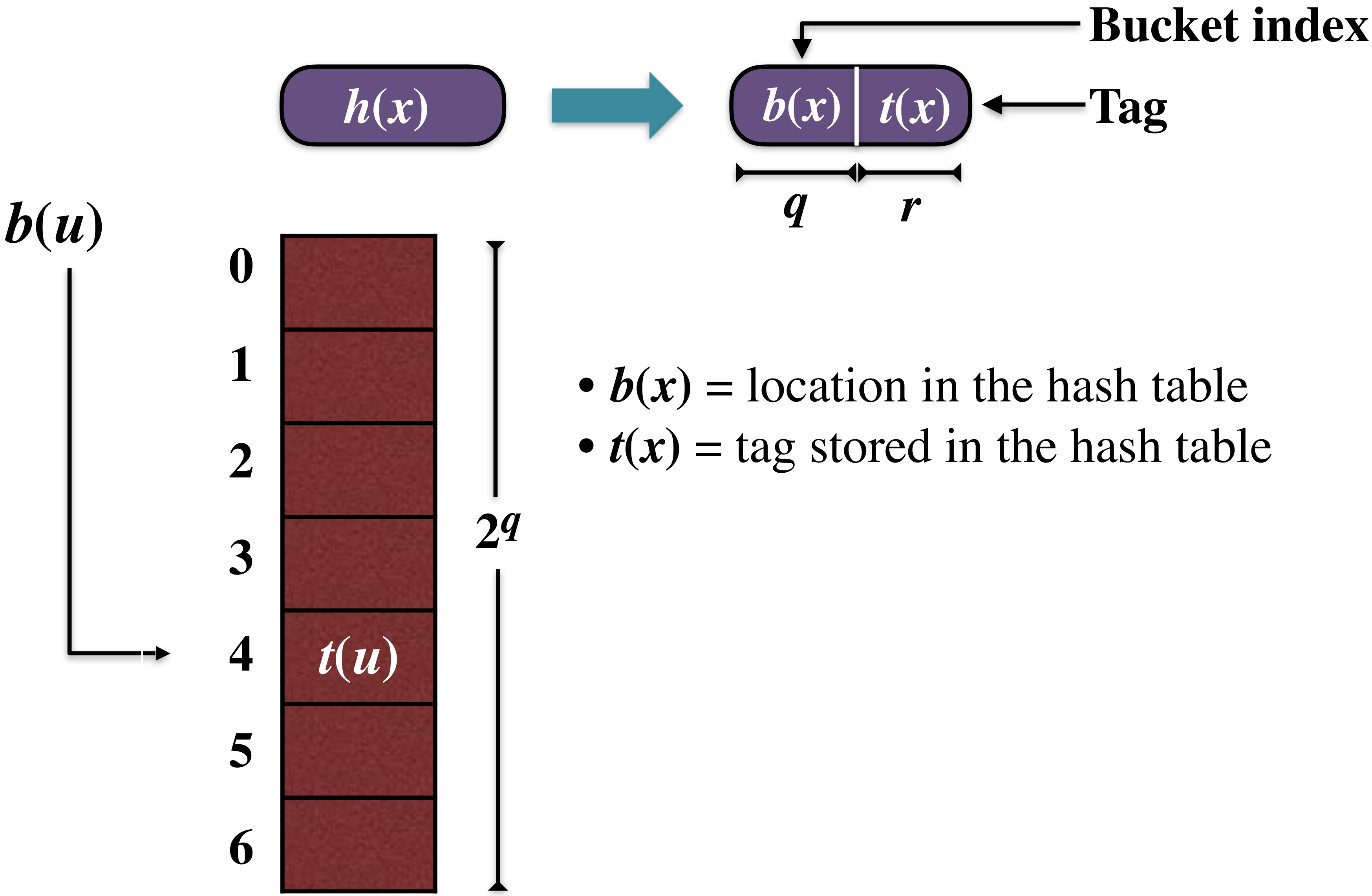
Quotienting: An alternative to Bloom filters

- **Store fingerprint compactly in a hash table.**
  - Take a fingerprint ***h(x)*** for each element ***x***.



- **Only source of false positives:**
  - Two distinct elements ***x*** and ***y***, where ***h(x)*** = ***h(y)***.
  - If ***x*** is stored and ***y*** isn't, ***query(y)*** gives a false positive.

Storing compact fingerprints



- $b(x)$ = location in the hash table
- $t(x)$ = tag stored in the hash table

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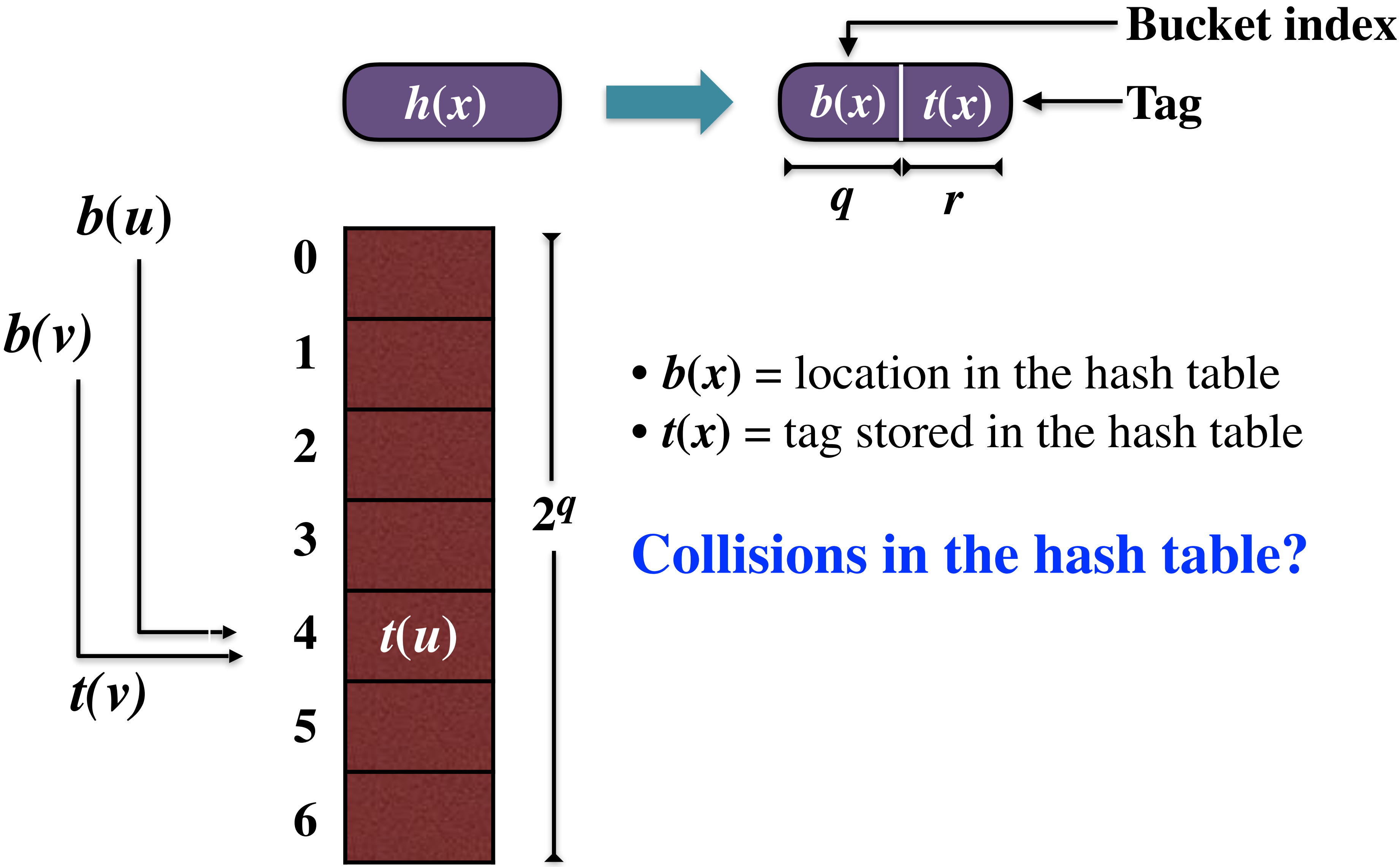

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Collisions in the hash table?

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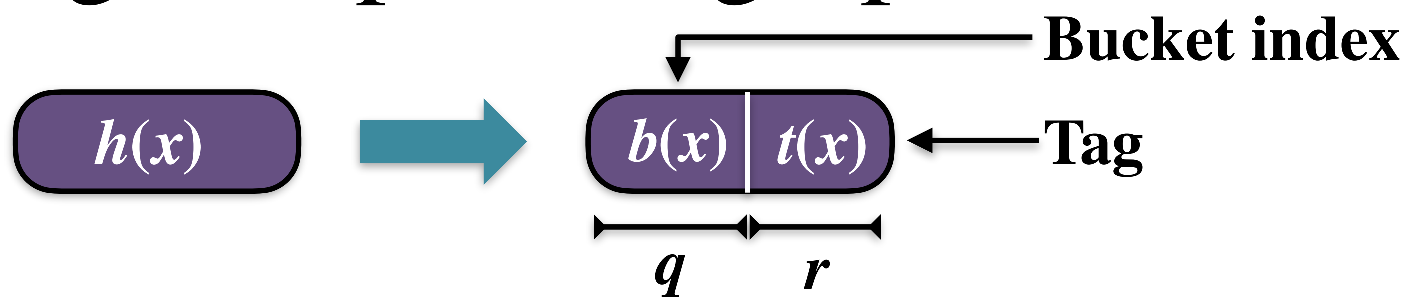

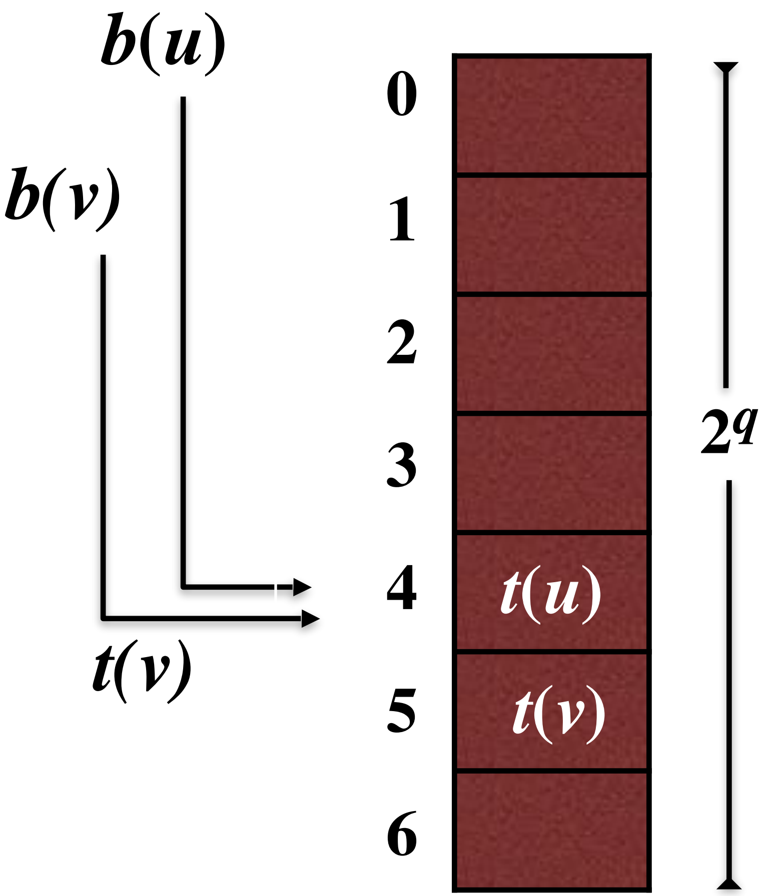

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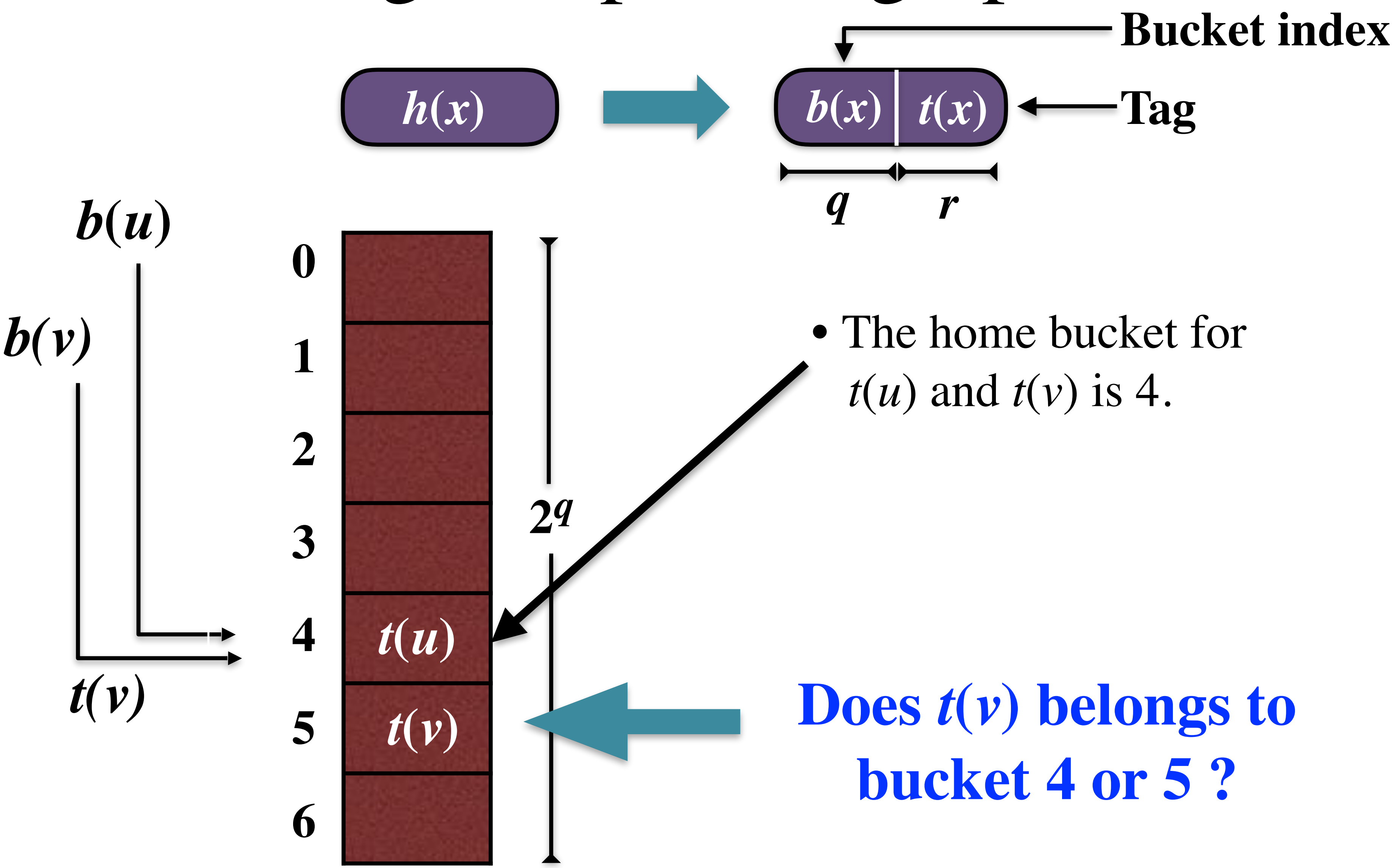
Collisions in the hash table?
Linear probing.

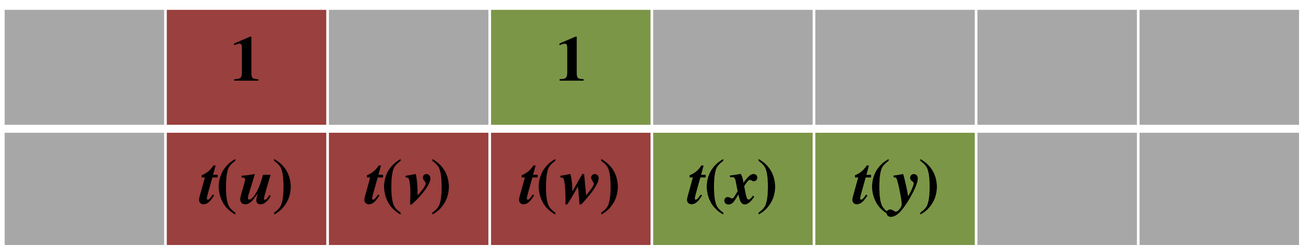
Storing compact fingerprints



Does $t(v)$ belongs to bucket 4 or 5 ?

Resolving collisions in the CQF

- CQF uses two metadata bits to resolve collisions and identify the home bucket.



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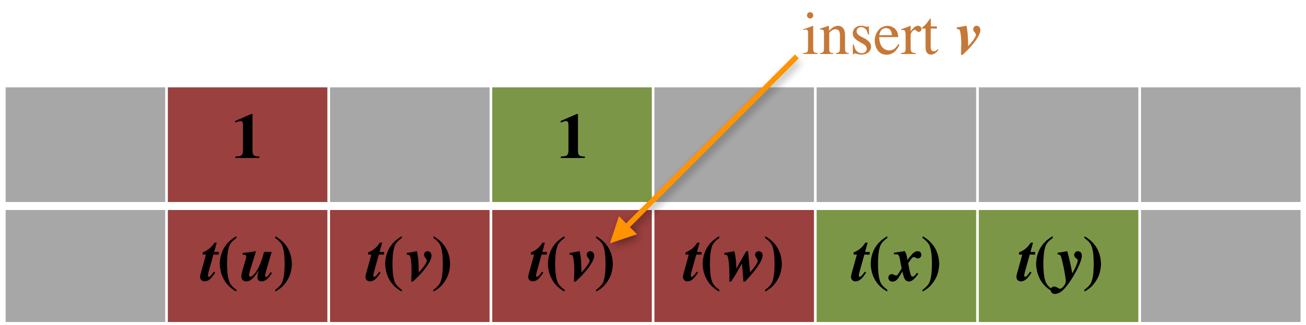

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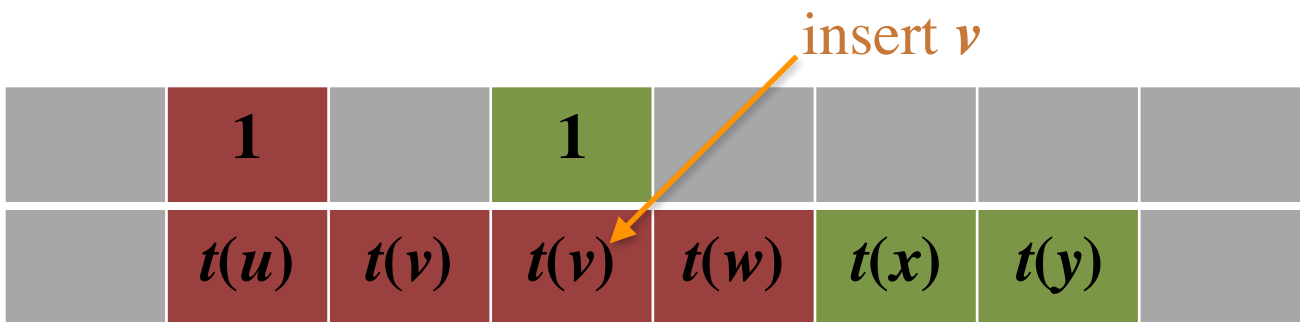


- The metadata bits group tags by their home bucket.

The metadata bits enable us to identify the slots holding the contents of each bucket.

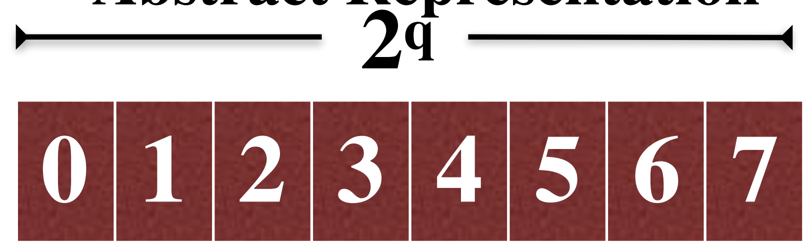
Counting quotient filter (CQF)

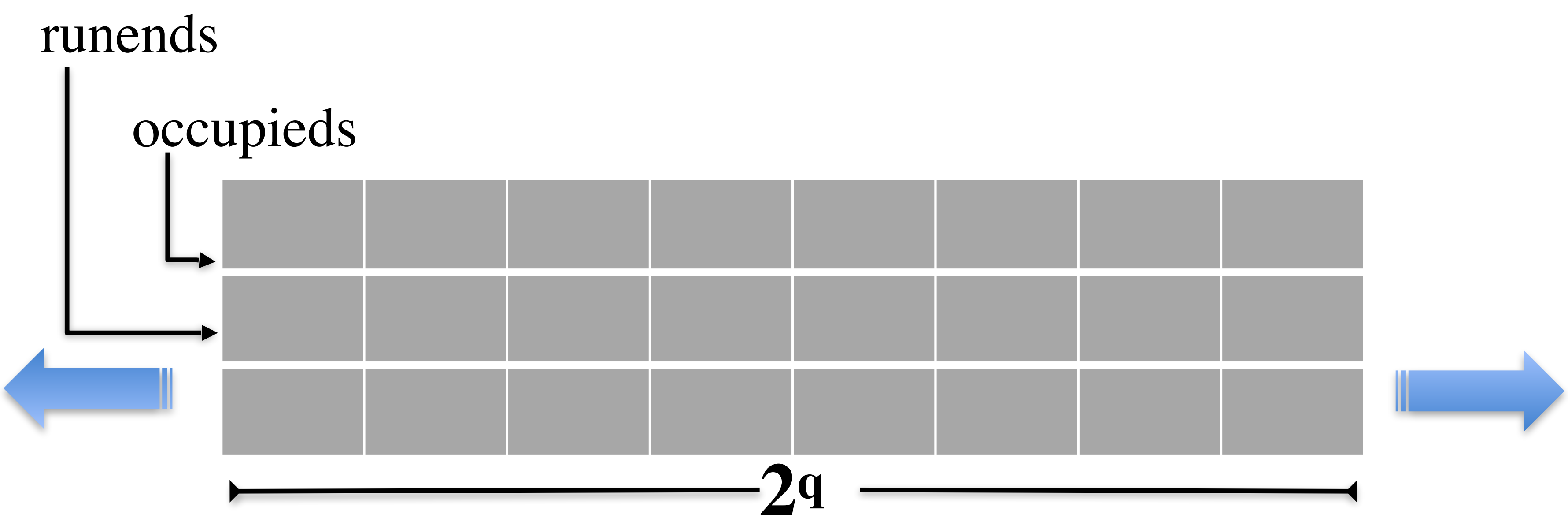
Implementation:

2 Meta-bits per slot.

h(x) --> h_0(x) || h_1(x)

Abstract Representation





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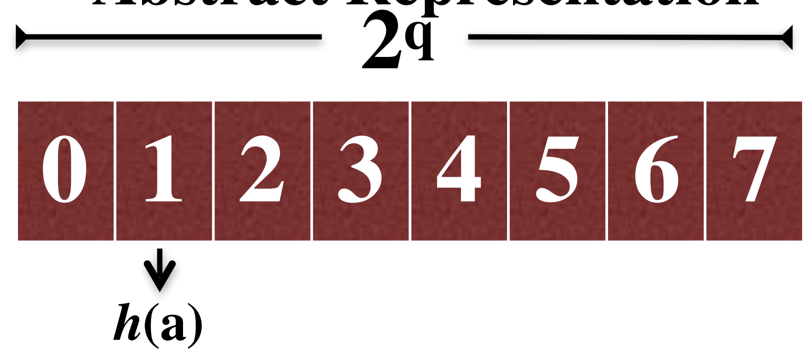

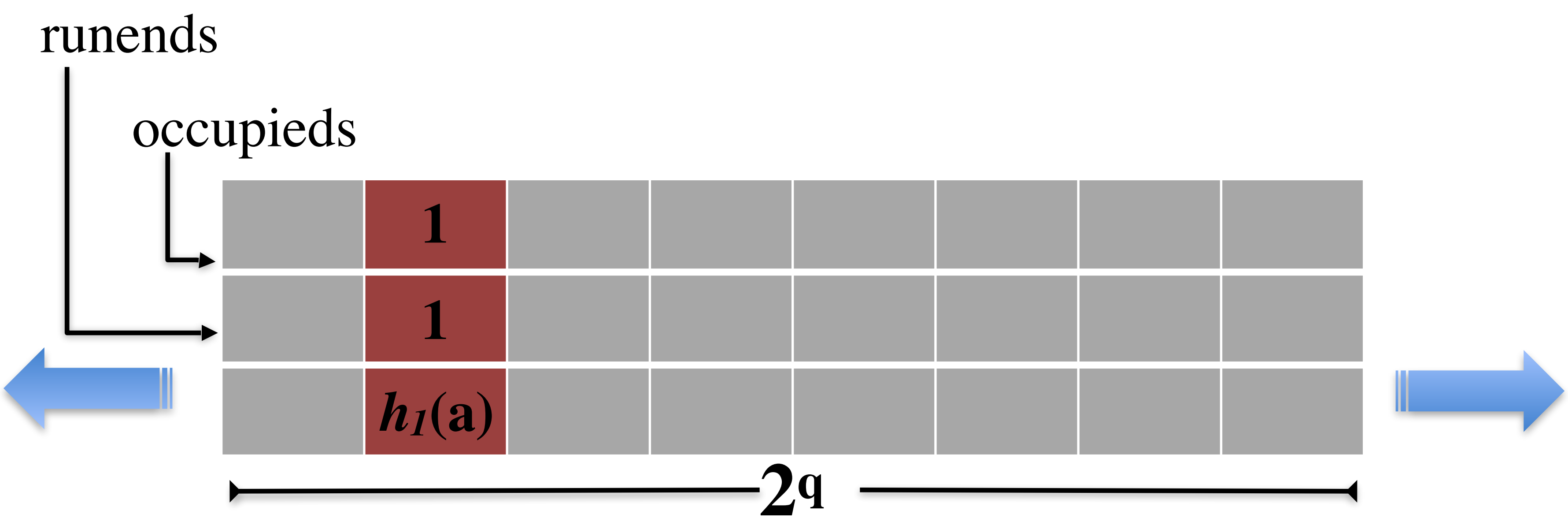

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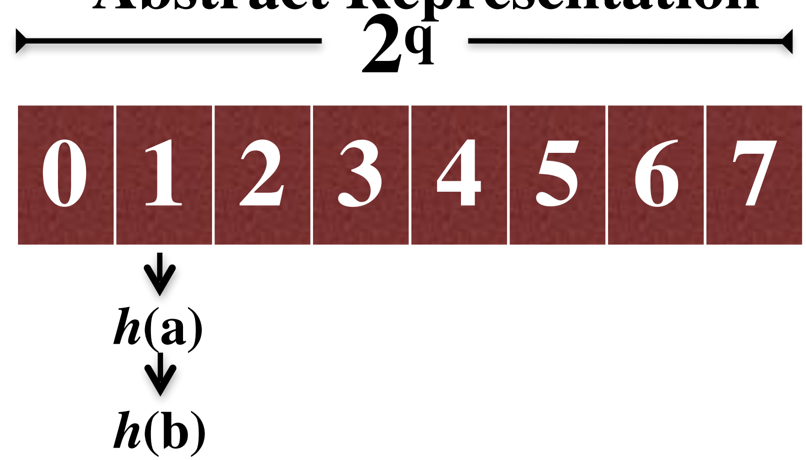

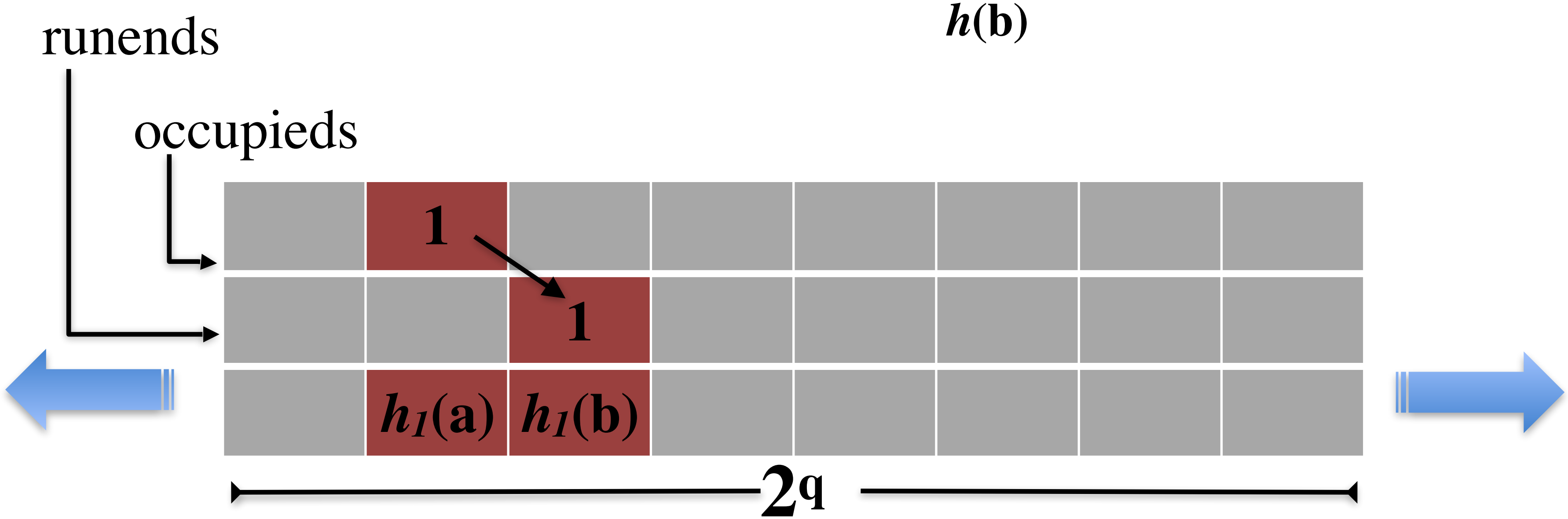

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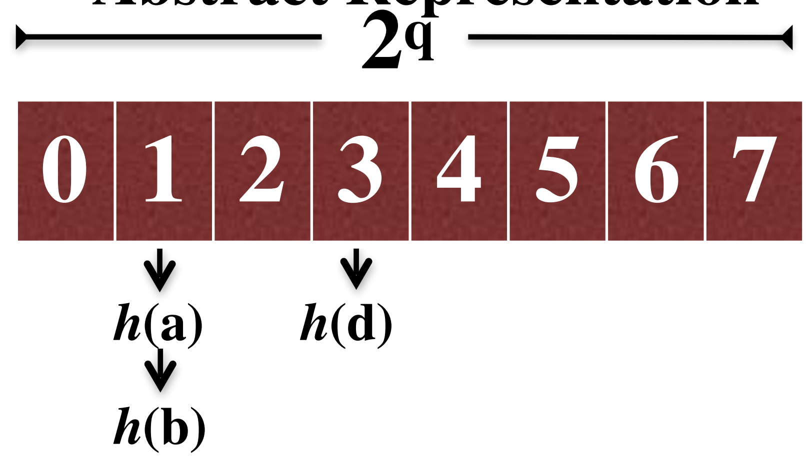

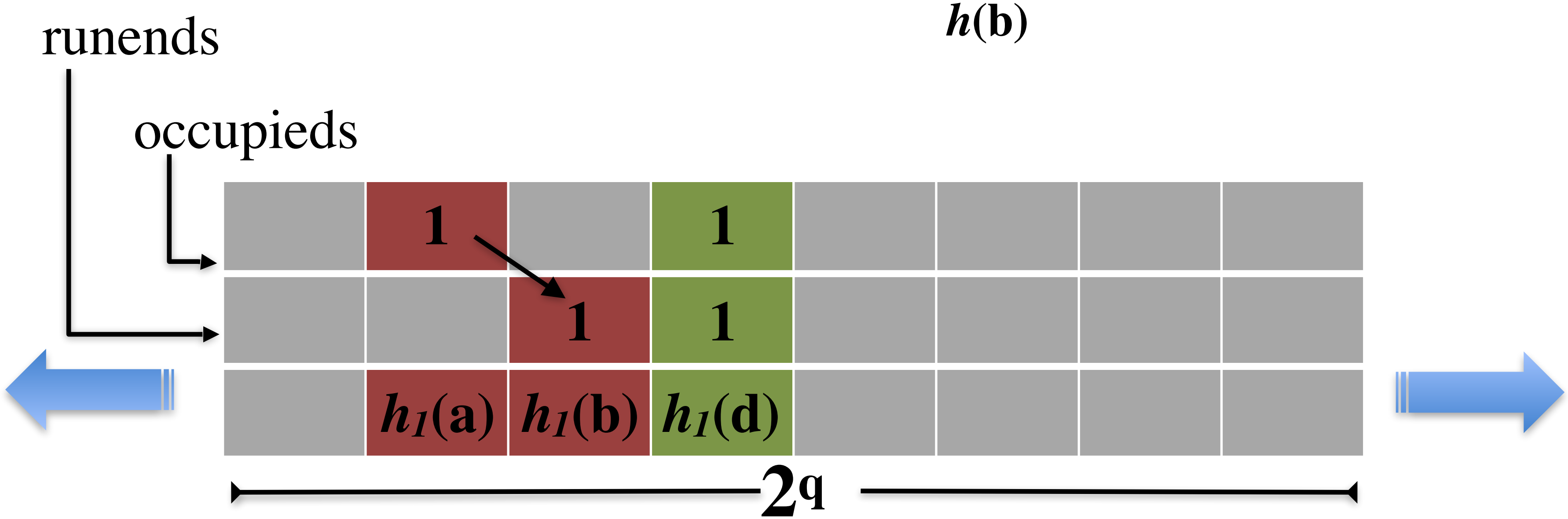

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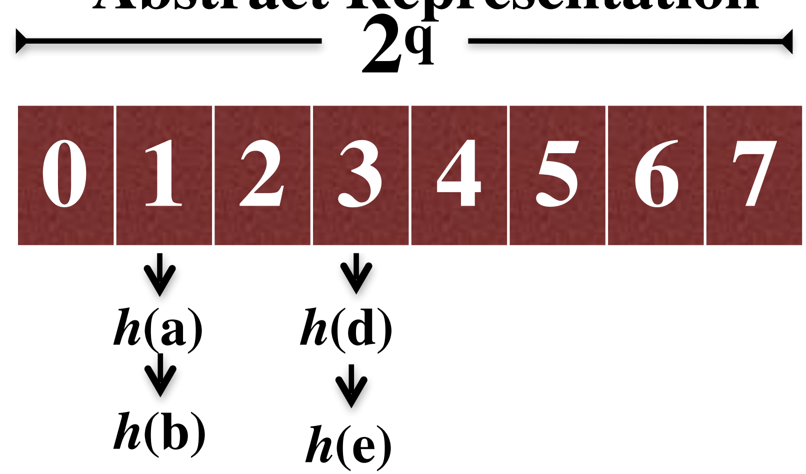

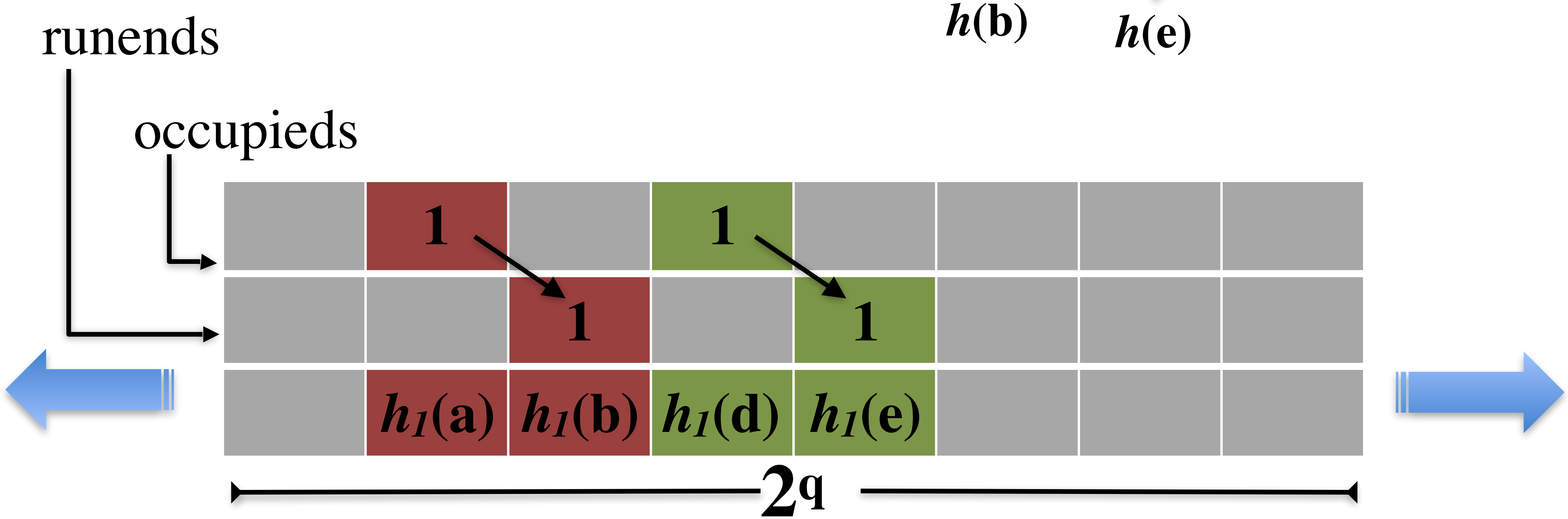

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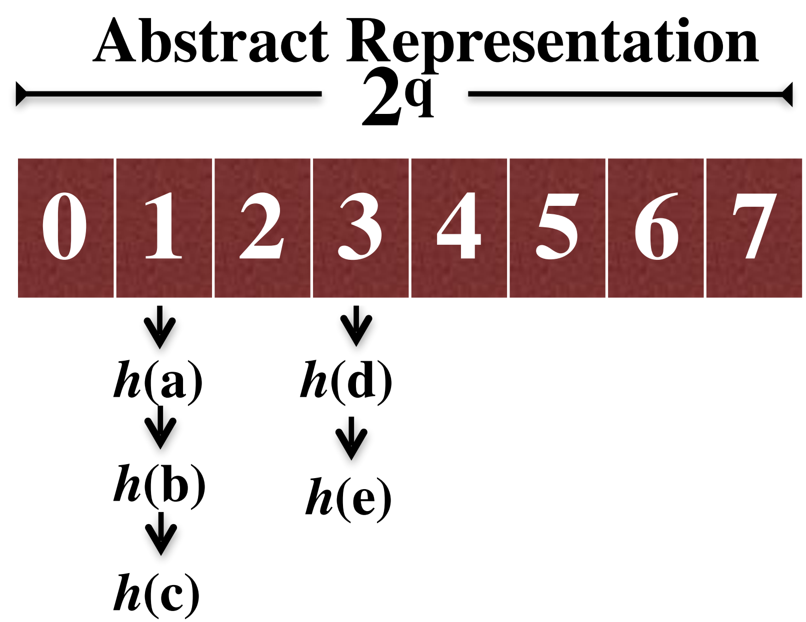

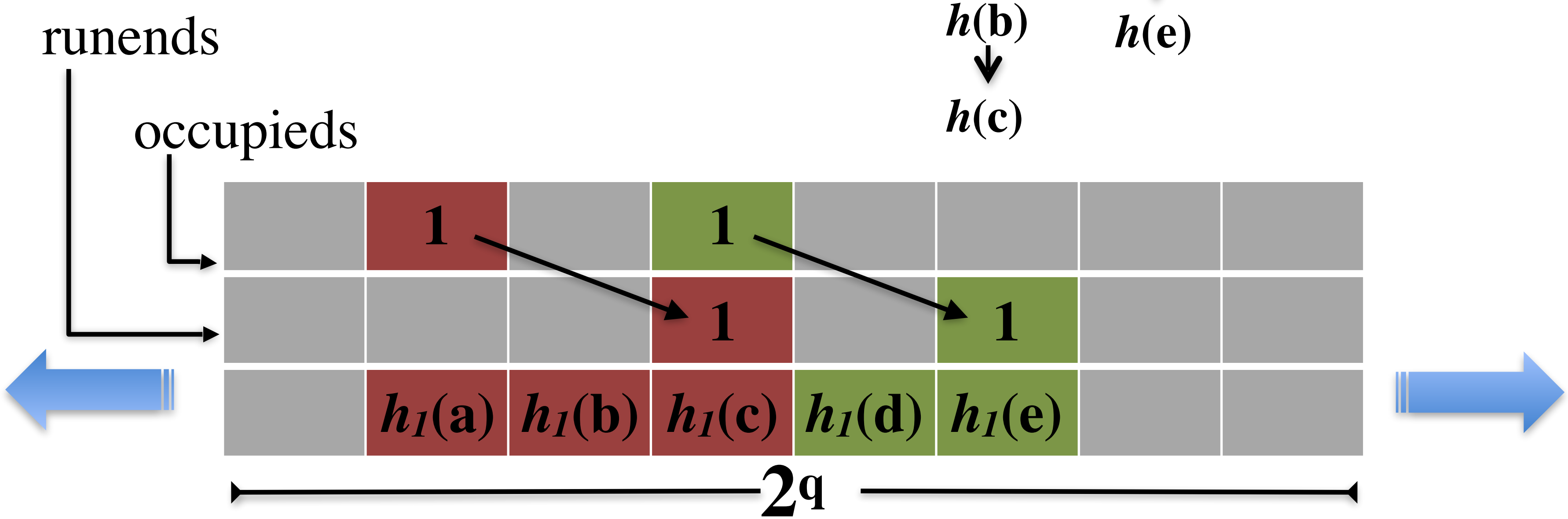

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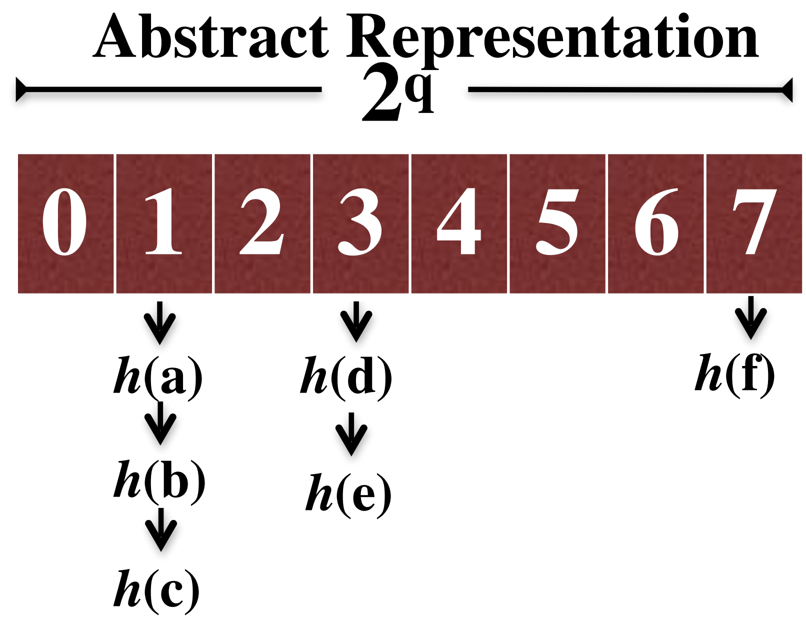
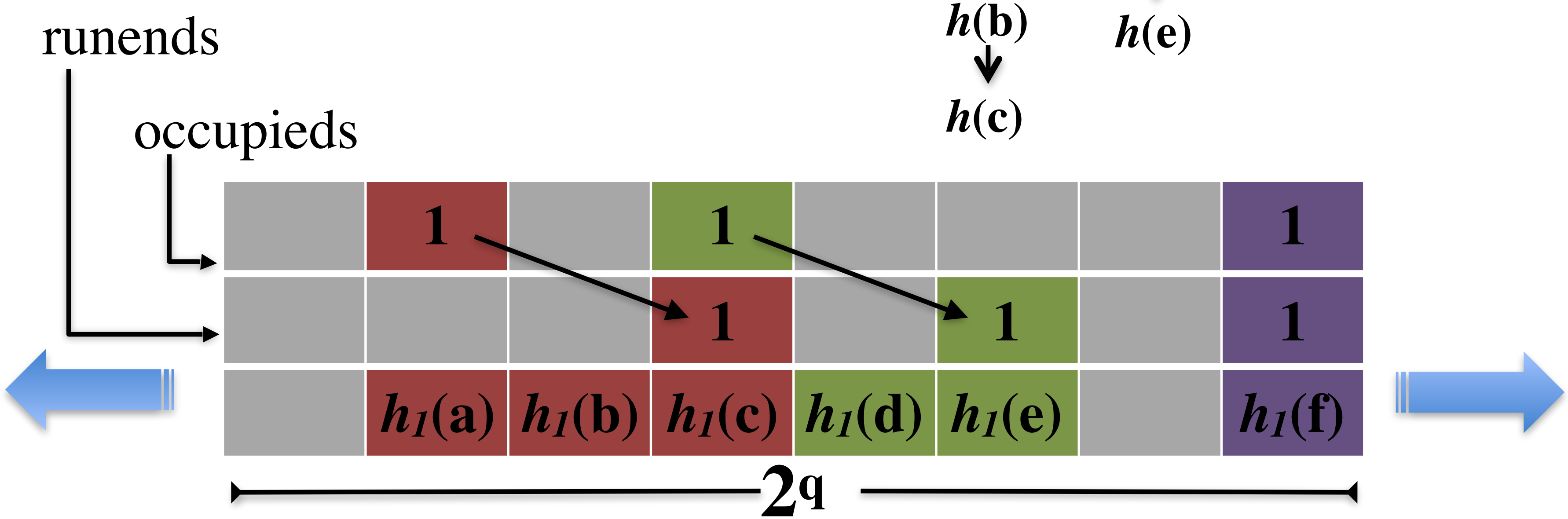
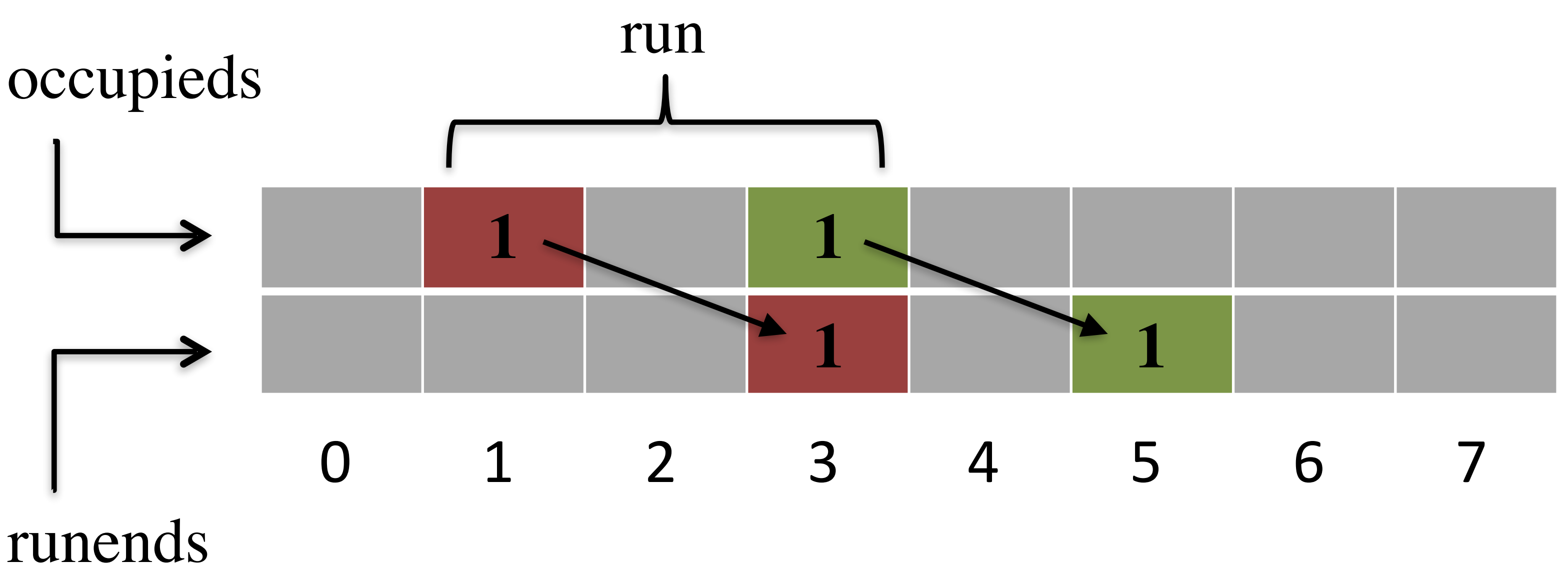




Metadata operations



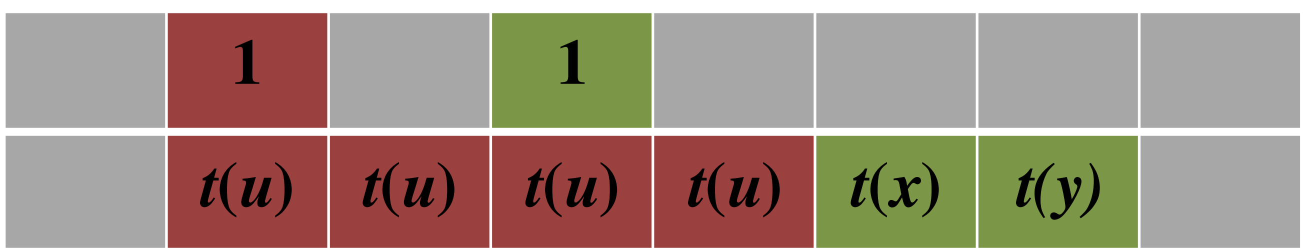
$$Rank(\text{occupieds}, 3) = \color{blue}{2} \qquad Select(\text{runends}, \color{blue}{2}) = 5$$

- Can accelerate metadata operations using x86 bit-manipulation instructions.
- Asymptotic improvement in query performance over the original QF.

Encoding counts

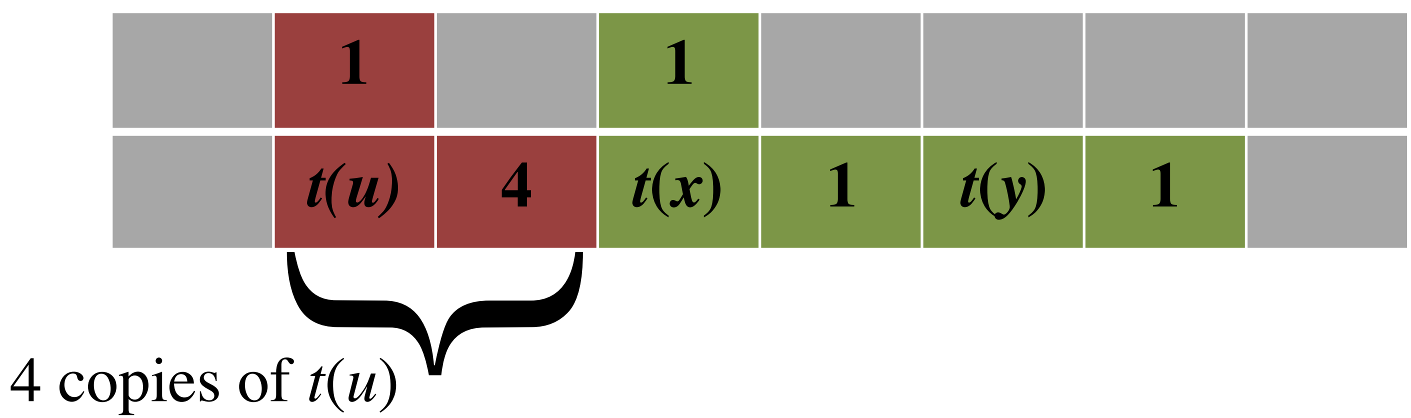
Encoding counts

- Metadata scheme tells us the run of slots holding contents of a bucket.
- We can encode contents of buckets however we want.
- ***The original quotient filter used repetition (unary).***



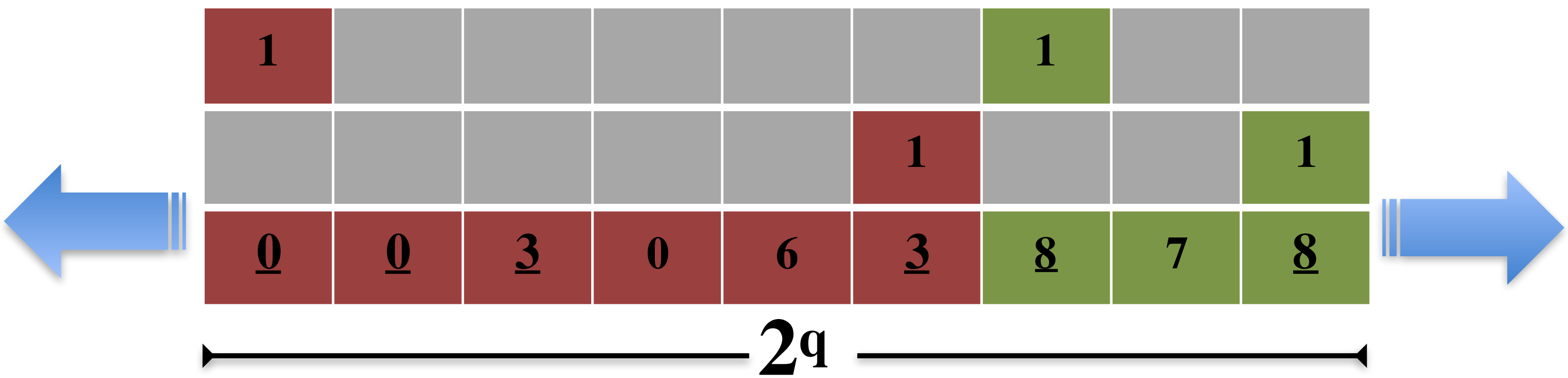
Encoding counts

- ***We want to count in binary, not unary.***
- Idea: use some of the space for tags to store counts.
- Issue: determine which are tags and which are counts without using even one “control” bit.



Encoding counts

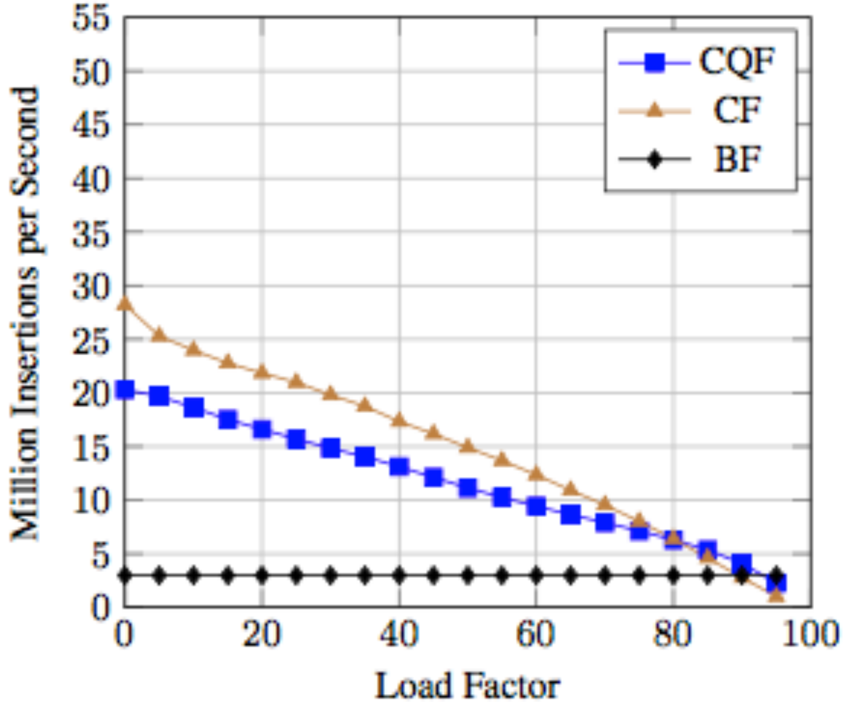
Dataset: 2 copies of 0, 7 copies of 3, and 9 copies 8.



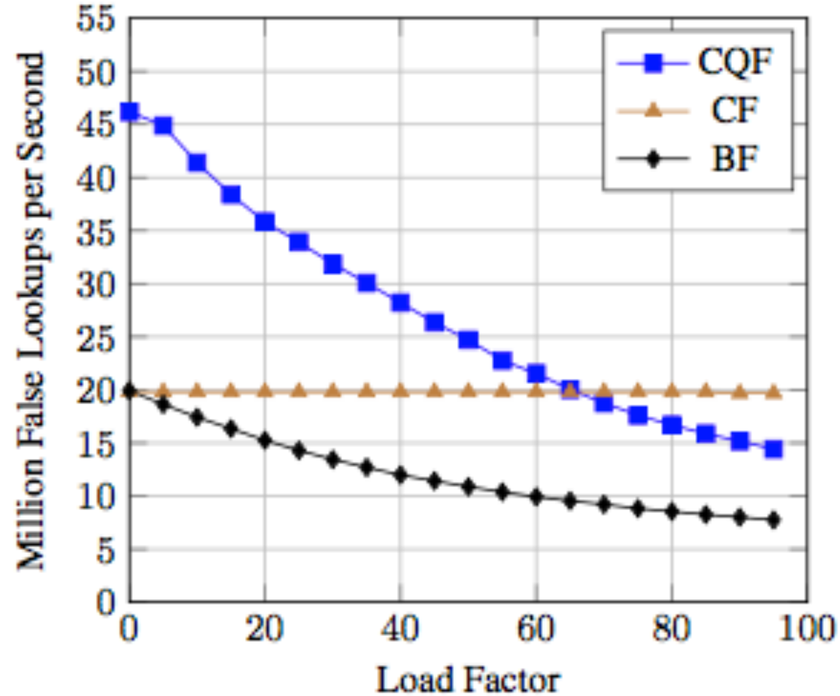
- An encoding scheme to count the multiplicity of items.
- Variable-sized counter.
- Using slots reserved for remainders to, instead, store count information.

Performance: In memory

Inserts

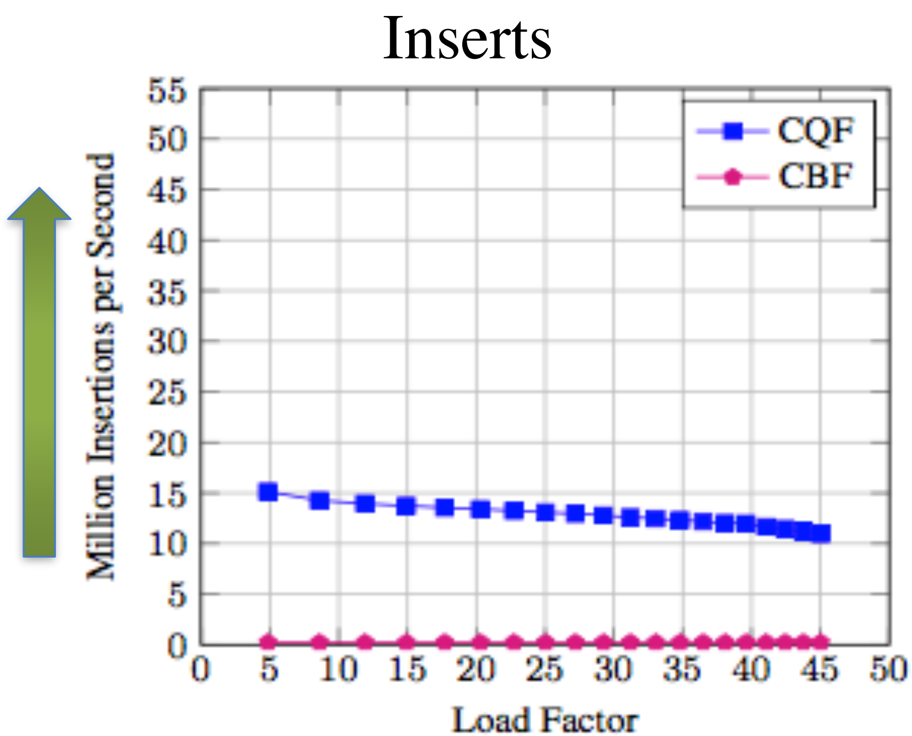


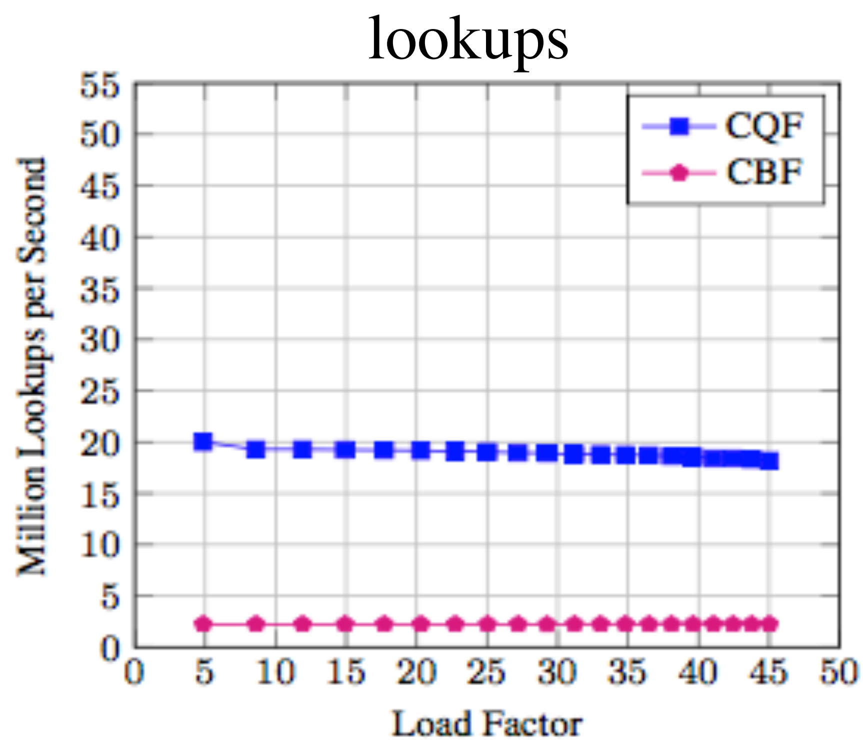
lookups



- The CQF insert performance in RAM is similar to that of state-of-the-art **non-counting** AMQs.
- The CQF is significantly faster at low load factors and slightly slower on high load factors.

Performance: Skewed datasets





- The CQF outperforms the CBF by a factor of 6x-10x on both inserts and lookups.

Conclusion

- The CQF is smaller and faster than other AMQs,
  - even ones that can't count.
- The CQF also supports deletes, resizing, cache locality, and other features applications need.
- The CQF demonstrates the extensible design of the quotient filter.

https://github.com/splatlab/cqf

Space analysis: Bloom Filter

- m = # of bits
- n = # of elements
- k = # of hash functions

- $k = m/(n \ln 2)$
- bits per element $S = m/n$
- false-positive rate = $2^{-m/(n \ln 2)} = 2^{-S \ln 2}$

Space analysis: Cuckoo Filter

- f = # of fingerprint bits
- b = # of entries in each bucket
- α = load factor

- bits per element $S = \alpha/f$
- false-positive rate = $2b/2^f = 2b/2^{S\alpha}$

Space analysis: Quotient filter

The quotient filter always takes less space than the cuckoo filter and offers better false-positive rate than the Bloom filter whenever $S \geq (c + \ln\alpha)/(\alpha - \ln 2)$

- bits per element $S = (r+c)/\alpha$
- false-positive rate $= \alpha 2^{-r} = \alpha 2^{-\alpha S + c}$